The Pseudo-Brewster Angle Revisited

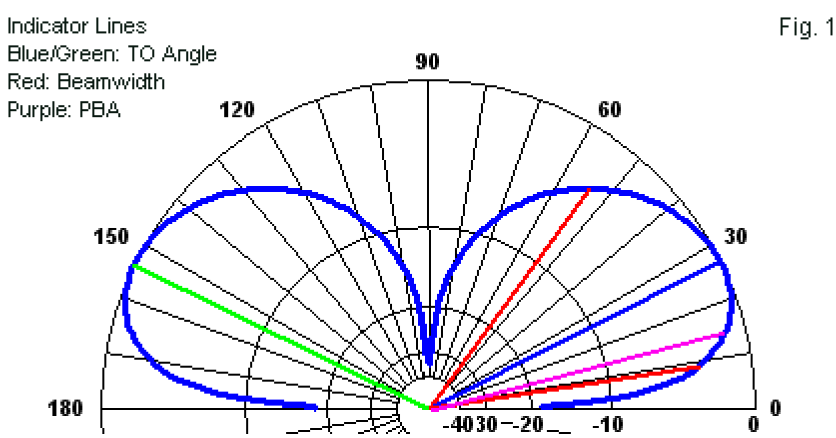
L. B. Cebik, W4RNL

One of the relatively unique features of *The ARRL Antenna Book* is Chapter 3, devoted to "The Effects of Ground." Too many radio amateurs tend to overlook this chapter as so much background on their way to the wire and aluminum assemblies that we call antennas. Within the wealth of useful data about the ground is a section drawn from the work of Charles Michaels, W7XC, on the pseudo-Brewster angle as it is applicable to vertical antennas.

Pseudo-Brewster-Angle Basics

Much of ground-reflection theory is drawn from optics on the premise that both light and radio waves are forms of electro-magnetic radiation. If we direct light upon a reflective surface, we obtain glare as the incident rays directly from the source and the reflected rays from the surface interact, that is, interfere with each other. In this context, the idea of interference includes both the case of one ray being out of phase with the other and therefore resulting in a weaker ray at the observer and the case where the two rays are in phase with each other and combine to form a stronger ray. However, there will always be an angle relative to the observer in which the phase angle between the incident and reflected ray is 90°. The result is a viewing angle without glare, that is, without interference effects between the incident and reflected rays. This is the Brewster angle, named after its 19th-century (1812) discoverer, Sir David Brewster.

Radio waves, especially those involving a vertically polarized antenna, display a very similar effect. The earth forms a lossy reflective surface, but one that reflects radio rays nevertheless. Hence, the energy transmitted (or received) is divisible into two components, the incident and the reflected rays. At a certain angle with respect to the horizon (assuming a flat earth), the incident and reflected rays will show a 90° phase difference, resulting a region of cross polarization. As Michaels notes, we call this the pseudo-Brewster angle (or PBA), since the action of RF energy mimics to a great degree (but not perfectly) the actions of light on reflective surfaces. The angle "of maximum polarization for the reflected light depends on the refractive index and hence varies with the wavelength and the kind of dielectric material used as the reflector." (The last line comes from an old college physics text on my shelf.)



Elevation Pattern for a 1/4-Wavelength Monopole with 64 Buried Radials
with Indicated Lines of Interest

Michaels 1987 treatment (*QST*, July, pp. 15-19) of the subject restricts itself to the effects of the pseudo-Brewster angle on one type of vertical antenna: the ground mounted 1/4-λ monopole with a suitable radial system beneath. Since Michaels wrote, we have become very familiar with the elevation patterns of such antennas, largely due to the widespread use of antenna modeling software. **Fig. 1** shows a typical pattern for a 7-MHz version of this antenna over average ground. The usual elevation pattern calculates the TO angle, that is, the elevation angle of maximum radiation. We find two such angles marked on the plot with blue and green lines. The blue line has surrounding red lines to mark the -3-dB power points that give us the conventional measure of beamwidth for the elevation lobe.

The diagram includes a final purple line that we normally do not find in the plots produced by NEC or by MININEC: the elevation angle of the PBA. Since the line does not correspond to any of the data that antenna-modeling programs offer, we have begun to forget the PBA. Michaels simply notes that below the PBA angle, the radiation of a ground-mounted vertical decreases rapidly, falling toward zero at the horizon. In a pattern taken over perfect ground, maximum gain would be at the horizon. In one sense, then, the PBA does some explanatory work in the overall account of why the far field of a vertical antenna goes to zero at zero-degrees elevation.

We may calculate the PBA solely from the ground quality in the reflection zone of the antenna and from the operating frequency. The antenna itself has no bearing on the PBA. As we shall soon discover, neither does the height of the antenna, even though the height may alter the way we look at the PBA. However, ground quality and frequency remain the key ingredients to establishing the PBA for vertical antennas. We normally (that is to say, within the growing conventionalization of the terms of antenna modeling software) encounter ground quality in the form of two separate terms, each of which is directly or indirectly measurable. One term is the ground conductivity (ρ), measured in siemens/meter (S/m). The other term is the relative permittivity (or dielectric constant) (ε) of the ground, where a value of 1 is the standard and applies to a vacuum or to dry air. Together, these terms define a refractive index and permit us to calculate the PBA.

The categorization of ground quality goes back to the 1930s. The soil descriptions in **Table 1** are commonly used in antenna modeling. The table represents an adaptation of values found in *The ARRL Antenna Book* (p. 3-13), which are themselves an adaptation of the table presented by Terman in *Radio Engineer's Handbook* (p. 709), taken from "Standards of Good Engineering Practice Concerning Standard Broadcast Stations," Federal Register (July 8, 1939), p. 2862. Terman's value for the conductivity of the worst soil listed is an order of magnitude lower than the value shown here.

If we ignore the two water entries, the trends in conductivity and relative permittivity coincide: as one decreases, so too does the other. The "Flat country" entry is an exception, as the conductivity continues to decrease, but the permittivity shows a very minor bump upward. The general impression that most newer modelers carry away from such tables as the one just shown is simple: the better the soil quality, the higher the gain for a ground mounted vertical antenna, indeed, for any vertical antenna. However, the last addition to the initially qualified statement is not strictly true. As we increase the antenna height and the operating frequency, we encounter point beyond which the reverse holds true. Poorer soil may show higher gains for high UHF antennas.

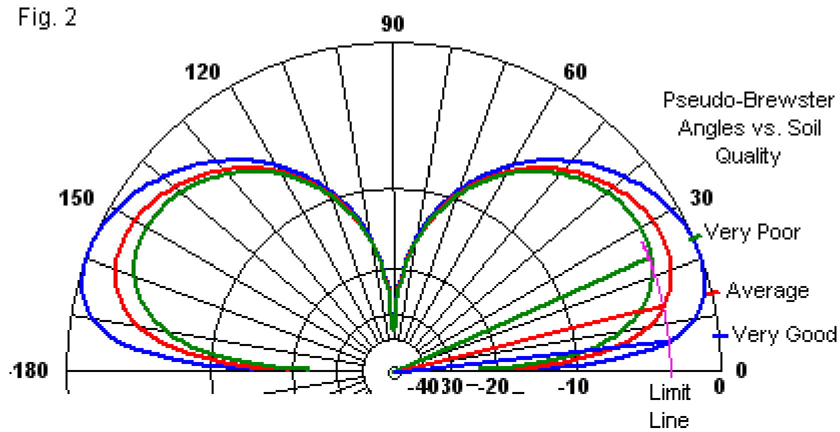
Table 1

Some Ground Quality Terminology and Values

Soil Description	**Conductivity in S/m σ**	**Permittivity (Dielectric Constant) ε**	**Relative Quality**
-Fresh water	0.001	80	
-Salt water	5.0	81	
-Pastoral, low hills, rich soil, typical from Dallas, TX, to Lincoln, NE	0.0303	20	Very Good
-Pastoral, low hills, rich soil, typical of OH and IL	0.01	14	Good
-Flat country, marshy, densely wooded, typical of LA near the Mississippi River	0.0075	12	
-Pastoral, medium hills, and forestation, typical of MD, PA, NY (exclusive of mountains and coastline)	0.006	13	
-Pastoral, medium hills, and forestation, heavy clay soils, typical of central VA	0.005	13	Average
-Rocky soil, steep hills, typically mountainous	0.002	12 - 14	Poor
-Sandy, dry, flat, coastal	0.002	10	
-Cities, industrial areas	0.001	5	Very Poor
-Cities, heavy industrial areas, high buildings	0.001	3	Extremely Poor

Perhaps the other generalization that modelers carry away from ground quality tables is that better ground quality normally yields a lower TO angle. **Fig. 2** overlays the elevation patterns for a 1/4-λ monopole with 64-buried radials. Although the plot does not produce the individual TO angle lines, we can easily see that the gain and elevation angle over very good soil (as defined in **Table 1**) are superior to the values we obtain for average and for very poor soil.

Fig. 2



Pseudo-Brewster Angle for a 1/4-Wavelength Ground-Mounted Monopole with 64 Buried Radials over 3 Levels of Ground/Soil Quality

At first sight, the 3 patterns in **Fig. 2** do not show any clear interrelationship. The following abbreviated data lines summarize the performance captured by the elevation plots.
Modeled performance of a 7-MHz 1/4-λ monopole with 64 buried radials

Ground Quality	Max. Gain DBi	TO Angle degrees	Half-Power Points degrees	Beamwidth degrees	PBA degrees
Very Good	1.76	21	6 / 50	44	6.4
Average	0.26	27	9 / 54	45	13.3
Very Poor	-0.59	30	11 / 57	46	23.2

On the plot, I have marked the calculated PBAs for the three soil quality levels. I have also drawn a concentric "limit" line, indicating where each PBA line intersects the pattern outline. Note that the lines form a single gain value (or very close to it). Allowing for small differences created by the increased losses of poorer soil at the antenna junction with ground, the PBA limit line marks the point in each pattern where the incident ray alone shows its gain level.

We might allow the PBA to fall into the background as an interesting but not very significant phenomenon if all that it indicated was a certain point below the TO angle for a ground-mounted vertical monopole. However, the PBA reappears in many ways that we often fail to recognize. Consider a vertical antenna (of any type) many wavelengths above ground. Let's arbitrarily pick a height of 10 λ, typical of many VHF and UHF antennas. When we model this situation, we shall use a plotting increment of 0.1° between elevation pattern sampling points to be certain that we obtain an accurate portrait of the lobes and nulls. Like the ground-mounted monopole, we shall sample the antenna over very good, average, and very poor ground. The results will look like the patterns in **Fig. 3**.

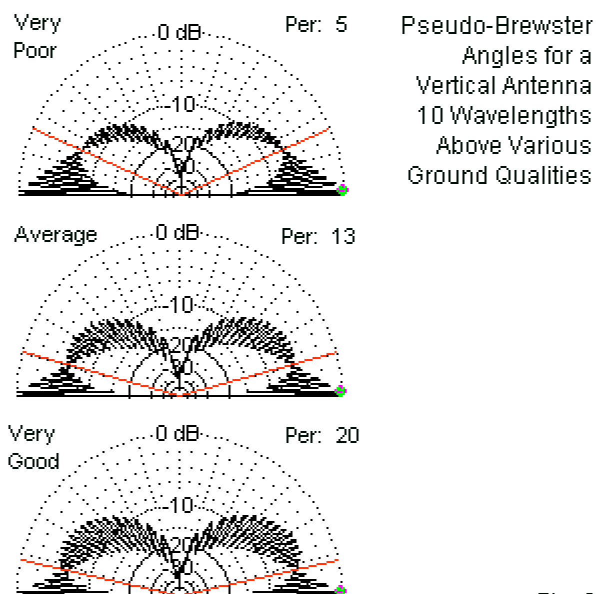


Fig. 3

Within each pattern, we find a region in which the strength of the lobes and depth of the nulls both decrease toward zero. If we draw a line from the pattern center through the point at which the lobes and nulls are minimum, we obtain a set of angles relative to the horizon, shown in red in the graphic. These lines decrease in angle as the soil quality improves. The angles shown are PBAs.

Perhaps the foremost surprise (at least for some) lies in the fact that these antennas show many lobes below the PBA elevation angle. In fact, for each pattern, the lowest lobe is also the strongest lobe, a desirable trait for antennas designed for point-to-point communications. If we had restricted ourselves solely to a discussion of ground-mounted monopoles, we might have missed that fact that the PBA is an angle at which we find only incident ray strength, but nothing else that is universal. Only if the elevation pattern has a single lobe with most of the energy above the PBA angle does the PBA mark a rapid decrease in gain toward zero. If the lowest elevation lobe is below the PBA, it can appear at full strength--although the gain will still go toward zero as we decrease the elevation angle of interest below the TO angle.

Calculating the PBA

The calculation of the PBA appears in the running text of *The Antenna Book* but in an appendix to the original 1987 article. The equations appear somewhat forbidding in their size, but we can clarify them with a little understanding of how they emerge.

Although we enter the ground quality into such programs as NEC or MININEC in the form of two separate values (conductivity and relative permittivity), ground calculations combine the inputs into a single value called the complex permittivity of the ground (k'). Here we shall use Michaels' designations of G for the conductivity and k for the permittivity, although most applications today would use ρ and ε.

$$k' = k - j\left(\frac{1.8 \cdot 10^4 \cdot G}{f}\right)$$

Note that the conductivity term requires an adjustment for the operating frequency (f), but the relative permittivity (k) does not. As well, in the calculation of the complex permittivity, we have a complex number (as indicated by the "j" or imaginary operator). Conductivity is the inverse of resistivity, which we measure in Ohms/m. Permittivity has an ultimate unit of Farads/meter. Hence, the two components are 90° out of phase with each other (just as are the components of a feedpoint impedance that might be 52 - j12 Ω). Combining the two into a magnitude requires standard vector addition techniques.

We might take a PBA calculation and pack into it all of the conductivity and permittivity components. However, many of the terms would require repetitious calculation, so Michaels' PBA calculation occurs in 2 steps. The first involves a calculation of a factor *x* based on the conductivity facet of the overall calculation:

$$x = \frac{1.8 \cdot 10^4 \cdot G}{f}$$

Once we have the value of *x*, we can plug it into a relatively straightforward calculation of the PBA. We shall expect and find a number of steps related to the sum of squares techniques that are part of the vector addition process.

$$PBA = \sin^{-1}\sqrt{\frac{k - 1 + \sqrt{(x^2 + k^2)^2 (k-1)^2 + x^2[(x^2 + k^2)^2 - 1]}}{(x^2 + k^2)^2 - 1}}$$

The equation is a fit subject for translation into a spreadsheet format in which we can pre-calculate various portions on the way to a final value. Note the number of occurrences of

$(x^2+k^2)^2$ and of (k-1). By breaking down the equation into repetitive parts, we can easily create a small spreadsheet or utility to perform the calculational drudgework.

Rapid calculation of the PBA allows us to set up any number of tables for studying the behavior of the PBA under various soil conditions at various operating frequencies. *The Antenna Book* has a small table of sample PBAs for 7, 14, and 21 MHz. I decided to expand the range of operating frequencies to include the US amateur bands from 160 m to 70 cm (with 300 MHz thrown in, since we often use that frequency for general design and analysis purposes). **Table 2** provides the calculated PBA value for these bands for most of the soil quality levels listed in **Table 1**.

Pseudo-Brewster Angles for Various Soils and Frequencies											Table 2
Soil->	Salt Wat	Fresh Wa	V Good	Good	G-	Av+	Ave	Poor	Poorer	V Poor	Ex Poor
Con/Per->	5/81	.001/80	.0303/20	.01/14	.0075/12	.006/13	.005/13	.002/12	.002/10	.001/5	.001/3
Freq MHz	PBA in degrees above horizon										
1.8	0.26	6.36	3.29	5.71	6.58	7.32	7.98	11.83	12.09	17.10	17.83
3.75	0.37	6.37	4.73	8.11	9.31	10.18	10.94	14.42	15.20	21.29	23.92
5.35	0.44	6.38	5.63	9.49	10.83	11.65	12.35	15.17	16.19	22.55	26.41
7	0.51	6.38	6.39	10.57	11.98	12.67	13.37	15.52	16.70	23.15	27.80
10.1	0.61	6.38	7.55	11.97	13.41	13.80	14.22	15.81	17.11	23.63	28.98
14	0.71	6.38	8.65	13.01	14.41	14.50	14.77	15.95	17.32	23.85	29.51
18.12	0.81	6.38	9.50	13.65	14.99	14.86	15.04	16.01	17.41	23.95	29.73
21	0.88	6.38	9.97	13.93	15.24	15.01	15.16	16.03	17.44	23.99	29.80
24.95	0.95	6.38	10.47	14.20	15.47	15.15	15.25	16.05	17.47	24.02	29.87
28	1.01	6.38	10.77	14.34	15.59	15.22	15.30	16.06	17.49	24.03	29.90
52	1.38	6.38	11.92	14.77	15.94	15.42	15.44	16.09	17.53	24.08	29.97
146	2.30	6.38	12.51	14.94	16.08	15.49	15.49	16.10	17.55	24.09	30.00
224	2.83	6.38	12.56	14.95	16.09	15.50	15.50	16.10	17.55	24.09	30.00
300	3.25	6.38	12.58	14.96	16.10	15.50	15.50	16.10	17.55	24.09	30.00
435	3.84	6.38	12.59	14.96	16.10	15.50	15.50	16.10	17.55	24.09	30.00
Est	*6.38*	*6.42*	*12.92*	*15.50*	*16.78*	*16.10*	*16.10*	*16.78*	*18.43*	*26.57*	*35.26*
Notes:	Soil Names: Salt Water; Fresh Water; Very Good; Good; Good -; Average +; Average; Poor; Pooer; Very Poor; and										
	Extremely Poor										
	Con/Per = conductivty in S/m and relative permittivity (dielectric constant)										
	Freq MHz = Sample frequency in MHz										
	Est = estimated PBA using simplified equation employing only the relative permittivity value										

On the assumption that we often only need an approximation of the PBA over dry land, the table also includes a frequency-independent simplified estimate of the elevation angle. Over dry land, conductivity values tend to be low, reducing the significance of that factor in the calculation. As well, as we increase the frequency of operation, the value of *x* goes down. By the middle of the HF range, the value of k (or ε), the relative permittivity, becomes the dominant factor in the calculation. Therefore, with due caution for its limitations, we may calculate an estimate of the PBA using only the relative permittivity.

$$PBA = \sin^{-1}\left(\frac{1}{\sqrt{\epsilon}}\right)$$

Table 2 places the estimated values of PBA in italics at the bottom of each column so that you may gather a sense of the conditions that make them usable or unusable as PBA values. As well you may see at what frequency for each listed soil quality the estimate begins to be accurate. For example, over salt water (conductivity 5 S/m), the estimate is useless. However, over fresh water (conductivity 0.001 S/m), the estimate is usable at all frequencies. Over average soil (conductivity 0.005 S/m, permittivity 13), the estimate is within a degree of the more accurate calculation from about the middle of the HF spectrum upward.

Graphing all of the columns would present us with a morass of overlapping lines that would defy any clarity. Therefore, **Fig. 4** graphs some of the major categories of ground quality. The X-axis is a log scale that--due to graphing limitations--extends beyond the upper and lower limits of the same. However, you may easily extrapolate the curves to both higher and lower values than those in the sample.

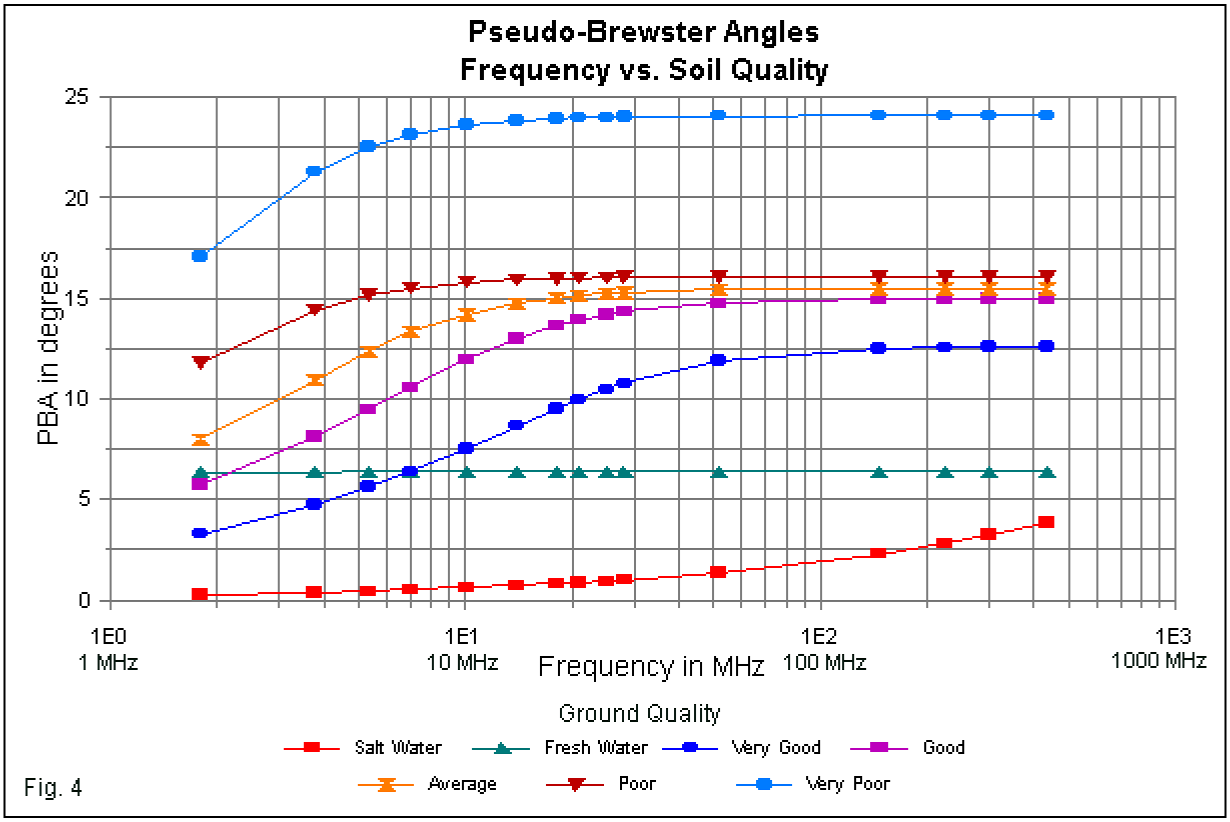


Fig. 4

The graph and table both make clear that the very high values of permittivity for salt and fresh water create PBA values whose curves are quite unlike the shapes of curves over dry land. Except for salt water and very good ground, the PBA values becomes nearly constant for a given soil quality above about 50 MHz. The slopes of the curves are relatively gradual down to about 8 MHz or so. Below that level, the PBA value decreases more significantly, especially with respect to the value produced by the simplified estimating equation.

The general dominance of permittivity in the PBA calculation shows up vividly in the table. Compare the columns labeled "Good," "G-," and "Ave+." The conductivity values show a steady decline: 0.01, 0.0075, 0.006. The permittivity values, however, follow this progression: 14, 12, 13. Although the G- column has a significantly higher conductivity than the Av+ column, the lower permittivity value yields consistently higher PBA angles. The differences may not be operationally significant, but they do illustrate the relative dominance of the main factors on PBA calculations.

Applications

The pseudo-Brewster angle has no very significant design applications in the field of antenna engineering. NEC software registers the angle as simply one calculation among many in the Sommerfeld-Norton ground calculation system. The resulting elevation patterns for vertical antennas simply show the consequences of the 90° phase-angle difference between the incident and reflected rays.

Perhaps the main function of knowing the PBA is that it goes a long way toward providing an explanation of some seemingly anomalous behavior in vertical antenna elevation patterns. Consider the patterns in **Fig. 5**. All of the patterns use the same vertical antenna, an elevated vertical dipole. The only difference among the 3 patterns is the height of the antenna feedpoint: 1 λ, 2 λ, and 5 λ above average ground.

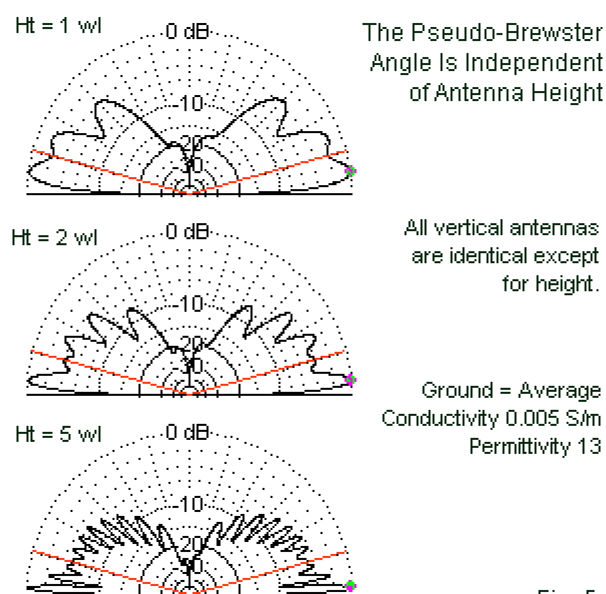
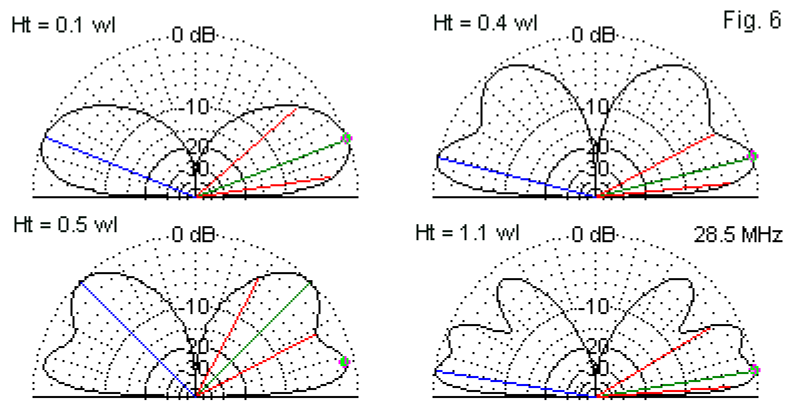


Fig. 5

If we take the first pattern in isolation, we might be struck by the shallowness of the null between the lower two lobes. The appearance would leave open the question of whether we had two lobes that simply are merging. Raising the antenna to a height of 2 λ does not fully answer the question. The weakness of the second elevation lobe and the shallowness of the nulls above and below it seem odd, especially when we compare them to elevation patterns for horizontal antennas that are less affected by the PBA. (See Michaels' treatment of the concept in "Horizontal Antennas and the Compound Reflection Coefficient," *The ARRL Antenna Compendium*, Vol. 3, pp. 175-184.) When we add the 5-λ-high version of the antenna, several matters begin to become clear. First, the normal null between lobes is quite deep, as evidenced by the high-angle nulls and the null between the first two lobes. Second, the diminution of both lobes and nulls occurs in the region of a single angle in all three patterns: the PBA. Third, the pseudo-Brewster angle does not change with the height of the antenna above ground, as the red indicator lines show. Fourth, elevated vertical antennas may have lobes that fall below the PBA.

PBAs, of course, do not explain everything. Consider a 1/4-λ monopole with 4 radials for 28.5 MHz. We shall place it a various heights above average ground: 0.1 λ, 0.4 λ, 0.5 λ, and 1.1 λ. **Fig. 6** shows us the resulting elevation patterns.



Elevation Patterns of a 1/4-Wavelength Monopole with 4 Radials at 28.5 MHz at Various Heights Above Average Ground

At the lowest antenna height, with the feedpoint only 0.1-λ above ground, we obtain a pattern not different in outline from the pattern for a ground-mounted monopole. The pattern itself does not show us the PBA, which for this frequency and ground type is about 15.2°. The maximum gain is 0.57 dBi at 21° elevation, while the gain at the PBA is 0.23 dBi. The only indication of the PBA is the continuous decline of gain below the PBA (and below the TO angle for that matter).

As we raise the base height of the monopole and its radials, a second elevation lobe emerges and grows. Secondary elevation lobes often have large beamwidths, as evidenced by the second plot in **Fig. 6**, taken for a base height of 0.4 λ. However, the highest gain for the antenna occurs at an elevation angle of 14° (0.93 dBi), just below the PBA. Although not clearly evident in the small-scale plot, the gain decreases less rapidly above the TO angle (in the direction toward the PBA) than below it. If we raise the antenna height to 0.5 λ, the second elevation lobe becomes the dominant lobe, with a maximum gain of 1.79 dBi at 45°. The lower lobe strength drops to 0.93 dBi at 13° elevation. This value is the same as the maximum gain for a height only 0.1-λ lower. The plot itself provides no direct evidence of whether the reversal of dominance between the lobes is or is not related to the polarization phenomenon, although we might note that we normally do not encounter such lobe reversals for 1/2-λ long horizontal dipoles as they generate secondary lobes with increased height above lossy ground.

(One clue to the function of the PBA phenomenon is the large rise in maximum gain as we raise the antenna by only 0.1 λ. One might intuitively sense that perhaps something is suppressing the gain of the lower elevation lobe. The source of the low rise in maximum gain for the lowest lobe should now be apparent.)

The lower lobe does not return to dominance until the antenna base height reaches about 1.1 λ, as shown by the last of the elevation plots. By this height, a third elevation lobe has appeared, and the angles of maximum gain for both lower lobes have correspondingly decreased. The lowest lobe is now well below the PBA. The gain is 2.79 dBi at 9°. Note the relatively high strength of the lowest lobe and its elevation angle compared to the gain value when the elevation angle was closer to the PBA. The height at which the lower lobe returns to domination is interesting because the shallow null between the first and second elevation lobes occurs just about at the PBA.

The PBA for vertical antennas, therefore, sometimes leaves the source of lobe strength unclear from a strictly visual perspective. Rarely do we take the time to analyze the gain levels at all sampled angles with sufficient detail to uncover PBA effects, and in most cases, such an analysis is unnecessary. The plot itself will tell us whether an antenna promises to provide satisfactory performance over the selected ground type.

Conclusion

Nonetheless, the pseudo-Brewster angle for vertical antennas remains an interesting and notable phenomenon of which we should be aware. In conjunction with other factors that influence the far field of an antenna, it helps us understand better the performance of vertical antennas over real ground, along with some of the differences that we obtain as we change the nature of the ground on which we install an antenna. It is a concept worth understanding down to the level of being able to calculate the angle itself--at least in approximate form. The approximation--while imprecise--does tell us where in a plot to expect to find effects of the PBA polarization phenomenon.